Quantum info in TCS: Homework 1

Guideline

- Deadline: October 4, midnight
- Each question is worth the same number of points. If your score is x out of 100, it will be rounded up to $\lceil x/20 \rceil * 20$.
- You should submit your solutions in groups of three or four members. Group assignments will be randomly selected and provided to you.

Problems

1. Prove that a quantum channel $\mathcal{N}_{A\to B}$ is entanglement-breaking (i.e., $(1_C \otimes \mathcal{N})(\rho_{CA})$ is a separable state for any bipartite density operator ρ_{CA}) if and only if its Choi state is a separable state.

2. Show that for any operator $A : \mathbb{C}^n \to \mathbb{C}^n$, the trace norm of A (i.e., $\|A\|_1$) is the solution to the following optimization:

$$\min_{W_1, W_2} \frac{1}{2}(\operatorname{tr} W_1 + \operatorname{tr} W_2) \tag{1}$$

$$\text{subject to } \begin{bmatrix} W_1 & A \\ A^\dagger & W_2 \end{bmatrix} \succeq 0 \tag{2}$$

   You can check that this optimization is a semidefinite program. What is it dual?

3. Let $|\Phi\rangle = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} |i, i\rangle$ be maximally entangled state and $n \geq 3$. If $\|\rho - |\Psi\rangle\langle\Psi|\|_1 \leq \frac{1}{3}$, then ρ is entangled.

4. Compute Holevo capacity of phase damping channel.

5-7. Exercise 2.4, 5.1, and 5.3 here.